Fluctuations of the String Tension and Transverse Mass Distribution

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Abstract

It is shown that Gaussian fluctuations of the string tension can account for the "thermal" distribution of transverse mass of particles created in decay of a colour string.

1. Recent precise data on the production rates of hadrons created in e^+e^- annihilation[1] allowed to analyse in detail the observed regularities. The main general conclusion from these analyses is that production of hadrons can be very well explained by a (rather unexpected) idea that they emerge from a thermodynamically equilibrated system.

This point was first recognized and emphasized some time ago by Becattini [1] who found that the particle spectra are consistent with the model of two thermally equilibrated fireballs. The temperature T determined from the fit was found to be about 160 MeV.

Recently Chliapnikov [2] analysed again the experimental particle distributions and found that they are also consistent with the thermal model[2].

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[1]A full list of data is given in [1, 2].
[2]I would like to thank B.Webber for calling my attention to this paper.

In his picture hadrons emerge from a thermally equilibrated system of (constituent) quarks and antiquarks. The fit to the data gives $T \approx 140$ MeV.

It should be emphasized that all this is achieved with at most 3 parameters, in contrast to the "standard" Monte Carlo codes like HERWIG and JETSET which are much less effective in this respect[3].

These findings are difficult to reconcile with the generally accepted ideas about hadron production in e^+e^- collisions. The main difficulty is that, since the process in question is expected to be rather fast, there is hardly enough time for any equilibrium to set in.

There is, however, a simple way to understand the findings of refs. [1, 2]: The spectrum of primarily produced partons (from which the final hadrons are formed) may be already so close to the thermal one that there is no further need for secondary collisions between partons to obtain the thermally equlibrated distribution (both life-time of the system and the parton density are irrelevant in this case). In the present note I shall argue that this possibility may naturally occur in the string model.

2. In the string picture of hadron production in e^+e^- annihilation [3, 4] the tranverse mass spectrum of the produced quarks (or diquarks) is taken from the Schwinger formula [5] which predicts [6]

$$\frac{dn_\kappa}{d^2p_\perp} \sim e^{-\pi m_\perp^2/\kappa^2} \tag{1}$$

where κ^2 is the string tension and $m_\perp$ is the transverse mass

$$m_\perp = \sqrt{p_\perp^2 + m^2}. \tag{2}$$

On the other hand, the "thermal" distribution is exponential in $m_\perp$

$$\frac{dn}{d^2p_\perp} \sim e^{-m_\perp/T} \tag{3}$$

rather than a Gaussian, as in (1).

The main point of the present note is the observation that Eq. (3) can be reconciled with the Schwinger formula (1) if the string tension undergoes fluctuations with the probability distribution in the Gaussian form

$$P(\kappa)d\kappa = \sqrt{\frac{2}{\pi < \kappa^2 >}} \exp\left(-\frac{\kappa^2}{2 < \kappa^2 >}\right) d\kappa \tag{4}$$

[3]A detailed discussion of this point can be found in [2]

where $<\kappa^2>$ is the *average* string tension, i.e.

$$<\kappa^2>=\int_0^\infty P(\kappa)\kappa^2 d\kappa. \tag{5}$$

Using (1) and (4) we thus have

$$\frac{dn}{d^2p_\perp} \sim \int_0^\infty d\kappa P(\kappa) e^{-\pi m_\perp^2/\kappa^2} = \frac{\sqrt{2}}{\sqrt{\pi <\kappa^2>}} \int_0^\infty d\kappa e^{-\frac{\kappa^2}{2<\kappa^2>}} e^{-\pi m_\perp^2/\kappa^2} \tag{6}$$

This integral can be evaluated using the identity [7]

$$\int_0^\infty dt e^{-st} \frac{u}{2\sqrt{\pi t^3}} e^{-\frac{u^2}{4t}} = e^{-u\sqrt{s}}. \tag{7}$$

The result is

$$\frac{dn}{d^2p_\perp} \sim \exp\left(-m_\perp \sqrt{\frac{2\pi}{<\kappa^2>}}\right) \tag{8}$$

i.e. the "thermal" formula (3) with

$$T=\sqrt{\frac{<\kappa^2>}{2\pi}}. \tag{9}$$

Using the standard value of the string tension $<\kappa^2>=0.9$ Gev/fm, we obtain $T=170$ MeV for the "temperature" of the primary partons, the value somewhat larger than those obtained by Beccatini [1] and Chliapnikov [2]. This is natural, as we expect that the primary parton system may undergo some expansion (and thus cooling) before the final hadrons start to form.

We thus conclude that the possibility of a fluctuating string tension may help to solve the apparent difficulty in the description of the mass and transverse momentum spectra in the string model. The nature of the fluctuations remains, however, an open question.

3. In search for a possible origin of such fluctuactions, it is tempting to relate them to stochastic picture of the QCD vacuum studied recently by the Heidelberg group [8, 9] [4]. In this approach the (average) string tension is given by the formula

$$<\kappa^2>=\frac{32\pi k}{81} G_2 a^2 \tag{10}$$

[4]It was shown that this picture helps to explain many features of the high-energy cross-sections [10].

where k is a constant ($k \approx .75$), G_2 is the gluon condensate [11] and a is the correlation length of the colour field in the vacuum (lattice calculations give $a = 0.35$ fm [12]). This result has a natural physical interpretation: It expresses the string tension (i.e. energy per unit lenght of the string) as a product of the vacuum energy density (proportional to the gluon condensate) times the transverse area of the string (proportional to a^2).

In the stochastic vacuum model both quantities entering the R.H.S of (10) are expected to fluctuate. Indeed, the gluon condensate G_2 is proportional to the square of the field strength (its average value can be estimated from studies of the charmonium spectrum [11]). Since the average value of the field strength in the vacuum must vanish, it cannot be constant but changes randomly from point to point. Also a^2 represents only the average value of the fluctuating transverse size of the string. Once this is accepted, it is natural to assume that such fluctuations are described by a Gaussian distribution. This implies fluctuations of the string tension in the form given by (4).

An interesting consequence follows from this point of view. First, the field fluctuations in the vacuum are expected to be independent at the two space-time points whose distance exceeds the correlation length a. This suggests that they may be also local *along the string*[5], although the corresponding correlation length a_s may be modified by the presence of the $q\bar{q}$ pair which creates the string (thus a_s may differ from a). This may have measurable effects. Indeed, observation of a heavy $q\bar{q}$ or baryon-antibaryon pair at a point along the string indicates that the string tension at this point took a value well above the average. Thus by triggering on heavy particles one may search for other effects of the large string tension, e.g. increasing multiplicity in the neighbourhood. Since any effect of this kind is limited to the region determined by the correlation length a_s, it may be possible to determine it experimentally and verify to what extent it is different from the vacuum correlation length a found in lattice calculation [12].

Needless to say, it would be very interesting to confirm or dismiss this picture using the lattice QCD. This, however, does not seem to be an easy task[6] .

Clearly, acceptance of the fluctuating string tension changes many other features of the string model. It is, however, beyond the scope of the present

[5] This was pointed out to me by W.Ochs.

[6] I would like to thank J.Wosiek for discussions about this point.

note to discuss them here.

4. In conclusion, we propose a modification of the original string picture by introducing a fluctuating string tension. We have shown that this assumption may help to explain the "thermal" nature of the spectra of particles produced in e^+e^- annihilation. We have also argued that it seems justifiable in the stochastic picture of the QCD vacuum. It remains an open and interesting question how this modification affects the successful phenomenology of the string model.

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